International Journal of Theoretical Physics, Vol. 33, No. 9, 1994

The Large Numbers Hypothesis and the Cosmological Constant

Aroonkumar Beesham[1]

Received March 1, 1994

A recent generalization of Dirac's large numbers hypothesis has implications for the cosmological constant problem. We show that this generalization follows from the usual large numbers hypothesis.

1. INTRODUCTION

Numerical coincidences have always attracted the attention of scientists. Most local dimensionless constants have values within an order of magnitude or so of unity. However, there exist a number of notable exceptions which have been highlighted over the years. In this paper we describe some of these large numbers and their possible implications for the cosmological constant.

2. LARGE NUMBERS HYPOTHESIS

Earlier in this century, Weyl, Eddington, and Milne noticed coincidences between some very large numbers that occur in nature [Barrow (1981, 1990), Barrow and Tipler (1986)]. This topic is usually associated with the name of Dirac (1937), who initially considered two large dimensionless numbers that could be constructed from fundamental constants and cosmological quantities.

The first was the ratio of the electric to the gravitational force between a proton and an electron

$$N_1 = \frac{e^2}{Gm_pm_e} \sim 10^{39} \tag{1}$$

[1]Department of Applied Mathematics, University of Zululand, Private Bag X1001, Kwa-Dlangezwa, 3886 South Africa.

The second was the age of the universe, expressed in atomic units (atomic light-crossing time)

$$N_2 = \frac{t_0}{e^2/(m_e c^3)} \sim 10^{39} \tag{2}$$

Several such large ratios can be constructed (Barrow, 1990). Based on such coincidences between such large numbers, Dirac (1938) put forward his so-called *Large Numbers Hypothesis* (LNH), which states:

- *Any two of the very large dimensionless numbers occurring in nature are connected by a simple mathematical relation in which the coefficients are of order unity.*

Since the number N_2 contains the age of the universe, an immediate consequence of the LNH is that any large number of the order 10^{40} must be equated to N_2, and hence must be time-dependent. Thus the LNH provides an explanation for the existence of large numbers of the order $(10^{39})^n$, $n = 1, 2, 3, \ldots$, as they are so large simply because the universe is as old as it is. To illustrate this idea further, consider the amount of matter in the visible universe, expressed in terms of the proton mass:

$$N = \frac{4\pi(ct)^3\rho}{3m_p} \sim \frac{c^3 t}{Gm_p} \sim 10^{78} \tag{3}$$

According to the LNH, this number must vary as t^2.

Now, from the LNH, the number N_1 must vary as t. Since a variation of e, m_p, or m_e would involve conflict with quantum physics, Dirac chose to consider instead a variation of G with time,

$$G \sim 1/t \tag{4}$$

The implications of such a proposal have been investigated in great detail [see the references in Barrow (1990) and Barrow and Tipler (1986)].

3. GENERALIZED LARGE NUMBERS HYPOTHESIS

Eddington (1935) found another large number which involved the cosmological constant

$$N_3 = \frac{c}{H_0}\left(\frac{m_p m_e}{\Lambda}\right)^{1/2} \sim 10^{39} \tag{5}$$

where H_0 is the present value of the Hubble parameter. The expression (5) is the ratio of the radius of curvature of de Sitter spacetime to the geometric mean of the electron and Compton wavelengths. Now, according to Dirac's LNH, the number N_3 increases with time as

$$N_3 \sim t \tag{6}$$

Since there was no reason for Dirac to believe in a variable Λ, the obvious conclusion was that Λ must vanish.

Instead of considering $\Lambda = 0$, Berman (1992) has proposed a generalized large numbers hypothesis (GLNH) which states that

$$N_1 \sim N_2 \sim N_3 \sim \sqrt{N} \sim t \tag{7}$$

From relations (5) and (7) it follows immediately that

$$\Lambda \sim 1/t^2 \tag{8}$$

The question that arises is whether it is really necessary to postulate a GLNH.

To answer this question, we note first that N_3 is a large dimensionless number which, according to the LNH, must vary as t. This means that either Λ must vanish, as pointed out by Dirac (1938), or that Λ must vary as in (8) (Barrow, 1990; Barrow and Tipler, 1986). Hence it is not necessary to elevate the status of relation (5) to that of a GLNH, since relation (5) is but a consequence of the LNH.

Second, the relationship (8) can also be derived from the LNH without recourse to the number in (5) (Lau, 1985). Consider Einstein's field equations (in suitable units) of general relativity with the cosmological term,

$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = G T_{ab} \tag{9}$$

where R_{ab} is the Ricci tensor, R the Ricci scalar, g_{ab} the metric tensor, and T_{ab} the energy-momentum tensor. Let us try to make these equations compatible with the LNH with the minimum amount of modification possible. The LNH requires that G be a function of time as in relation (4). Now the Bianchi identities and the divergenceless of the energy-momentum tensor are, respectively,

$$\left(R^{ab} - \frac{1}{2} R g^{ab}\right)_{;b} = 0 \tag{10}$$

$$T^{ab}{}_{;b} = 0 \tag{11}$$

From equations (9)–(11) we obtain

$$\Lambda_{;b} g^{ab} = G_{;b} T^{ab} \tag{12}$$

Since G is a function of time, it cannot have zero divergence, and thus equation (12) implies that Λ cannot be constant. Thus Dirac's LNH is not compatible with Einstein's general relativity. The simplest assumption that we can make about Λ is that it is a scalar function of time, and it only remains to determine its actual time dependence.

Let us consider the perfect fluid form for the energy-momentum tensor,

$$T_{ab} = (\rho + p)u_a u_b + p g_{ab} \tag{13}$$

where ρ is the energy density, p the pressure, and u_a the four-velocity of the fluid. In cosmology, it is always possible to choose $g_{00} = -1$. Then, taking the "00" component of equation (13), we obtain

$$T^{00} = \rho \tag{14}$$

Thus equation (12) yields

$$\dot{\Lambda} = -\rho \dot{G} \tag{15}$$

where the overdot denotes a derivative with respect to time.

We now only need the form of the energy density ρ to determine the time dependence of Λ. This may also be derived from the LNH as follows (Dirac, 1938; 1979; Lau, 1985). Assuming that mass is conserved [Dirac (1938) also considered the case when mass is not conserved, but later abandoned this idea (Dirac, 1979)], we obtain

$$\rho R^3 = \text{const}$$

Dirac (1979) then considered the general expansion of the universe to be given by

$$R \propto t^n$$

Take a particular galaxy whose velocity of recession is 1/2 (in units in which the speed of light is unity). This may be written as

$$\dot{R} = \frac{nR}{t} = \frac{1}{2}$$

Hence the distance of the galaxy from us is $t/(2n)$, so the total mass within this distance is proportional to ρt^3. By the LNH (3), this number must vary as t^2, and we then have

$$\rho t^3 \propto t^2$$

or the required result

$$\rho \propto \frac{1}{t} \tag{16}$$

From (4), (15), and (16), we then obtain our desired result,

$$\Lambda \sim \frac{1}{t^2} \tag{17}$$

It is interesting to note that this time dependence of Λ provides a phenomenological solution to the cosmological constant problem (Berman, 1992, and references therein). Λ is so small now simply because the universe is as old as it is.

4. CONCLUSION

In this paper, we have explained the LNH and some of its implications for cosmology, in particular for the cosmological constant problem. We conclude that it is not necessary to postulate a GLNH, as the so-called generalization follows directly from the LNH and Einstein's gravitational equations.

REFERENCES

Barrow, J. D. (1981). *Quarterly Journal of the Royal Astronomical Society*, **22**, 388–420.

Barrow, J. D. (1990). The mysterious lore of large numbers, in *Modern Cosmology in Retrospect*, B. Bertotti, R. Balbinot, S. Bergia, and A. Messina, eds., Cambridge University Press, Cambridge, pp. 67–93.

Barrow, J. D., and Tipler, F. J. (1986). *The Anthropic Cosmological Principle*, Oxford University Press, Oxford, pp. 219–287.

Berman, M. S. (1992). *International Journal of Theoretical Physics*, **31**, 1217–1219.

Dirac, P. A. M. (1937). *Nature*, **139**, 323.

Dirac, P. A. M. (1938). *Proceedings of the Royal Society*, **165**, 199–208.

Dirac, P. A. M. (1979). *Proceedings of the Royal Society of London A*, **365**, 19–30.

Eddington, A. S. (1935). *New Pathways in Science*, Cambridge University Press, Cambridge.

Lau, Y.-K. (1985). *Australian Journal of Physics*, **38**, 547–553.